Name: _____     Date: _____

# Student Exploration: Inclined Plane – Simple Machine

**Vocabulary:** coefficient of friction, efficiency, force, free-body diagram, friction, inclined plane, mechanical advantage, mechanical energy, normal force, resultant force, simple machine, vector, work, work-energy theorem

**Prior Knowledge Questions** (Do these BEFORE using the Gizmo.)
Jan is moving to a new apartment. She needs to load her sofa and other large furniture into a moving van. The rear of the moving van is 1.5 meters high.

1. What could Jan use to make loading furniture on the van easier? _____

   _____

2. Why would this help? _____

   _____

**Gizmo Warm-up**
A **simple machine** can be used to make tasks like lifting heavy weights easier. One example of a simple machine is a ramp, or **inclined plane**. You can use the *Inclined Plane – Simple Machine* Gizmo™ to see how inclined planes can help to lift objects.
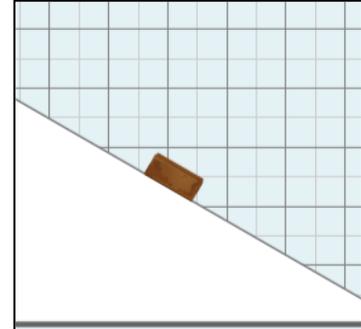
On the CONTROLS pane, make sure the **Angle** is 30°, the **Coeff. of friction** is 0.00, and the **Weight** is 300 N.



1. The brick has a weight of 300 newtons (N).

   How much force would it take to lift the brick straight up? _____

2. Set the **External force** to **On**. A car appears, ready to push on the brick. Set the **Applied force** to 100 N and click **Play** ( ▶ ). What happens?

   _____

3. Click **Reset** ( ↺ ). Using the Gizmo, find the smallest force that is required to push the block up the 30° ramp.

   What is the smallest force required? _____

| Activity A: | Get the Gizmo ready: | |
|---|---|---|
| **Redirection of force** | • Turn **Off** the **External force**. Click **Reset**.<br>• Set the **Angle** to 30° and the **Weight** to 300 N. | |
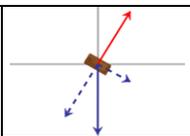
**Question: How does an inclined plane redirect a force?**

1. <u>Observe</u>: Select the FREE-BODY DIAGRAM tab. Make sure **Magnitude** is on. A **free-body diagram** is a picture that uses **vectors** to show the different **forces** acting on an object.

   What does the purple arrow pointing down represent? _____

   The inclined plane breaks this force down into two components: one parallel to the inclined plane (**W$_{\parallel}$**) and one perpendicular to the inclined plane (**W$_{\perp}$**).

2. <u>Infer</u>: Which force (**W$_{\parallel}$** or **W$_{\perp}$**) will cause the brick to slide down the plane? _____

3. <u>Calculate</u>: To calculate a ratio, divide the two numbers being compared.

   A. What is the ratio of $W_{\parallel}$ to the **Weight** of the brick? _____

   B. What is the ratio of $W_{\perp}$ to the **Weight** of the brick? _____

   C. Sine (sin), cosine (cos), and tangent (tan) are ratios of the lengths of a right triangle's sides. Use a calculator to find the sin, cos, and tan of the inclined plane's **Angle**.

   Sin: _____     Cos: _____     Tan: _____

4. <u>Synthesize</u>: Describe any relationships you see between the ratios you calculated and the sine, cosine, and/or tangent of the inclined plane's angle.

   _____

   _____

5. <u>Make a rule</u>: Use the relationships you found to write a formula for $W_{\parallel}$ and $W_{\perp}$ in terms of weight (*W*) and angle (*θ*):

   $W_{\perp}$ = _____          $W_{\parallel}$ = _____

6. <u>Apply</u>: If the brick's weight is 500 N and the plane's angle is 40°, what will $W_{\parallel}$ and $W_{\perp}$ be? Use the Gizmo to check your answer.

   $W_{\perp}$ = _____          $W_{\parallel}$ = _____

**(Activity A continued on next page)**

**Activity A (continued from previous page)**

7. <u>Solve</u>: As the weight of the brick pushes down on the inclined plane, the inclined plane pushes up against the brick. This upward force is called the **normal force**.

   A. What is the relationship between the normal force and $W_\perp$? _____

   _____

   B. The net force on the brick is the **resultant force**. What is its value? _____

   C. What force is equal to the resultant force? _____

   D. Select the SIMULATION tab. What is the mass of the brick? _____

   E. The formula for force is: *Force = mass × acceleration.* Use this formula to calculate

   what the brick's acceleration should be: _____

   Use the Gizmo to check your answer. (Click **Play**, and then select the TABLE tab and look at the **a (m/s²)** column. Downward accelerations are negative.)

8. <u>Make connections</u>: Click **Reset**. On the CONTROLS tab, switch the **External force** to **On**. Set the **Applied force** to 100 N. Select the FREE-BODY DIAGRAM tab. The green vector represents the force the car exerts on the brick.

   A. How does the direction of the applied force compare to the direction of $W_{||}$?

   _____

   B. Is the applied force enough to push the brick up the ramp? Explain. _____

   _____

   C. What is the minimum applied force needed to push the brick up the ramp? _____

   D. Use the Gizmo to check your answer. What applied force did you use? _____

9. <u>Apply</u>: Suppose you needed to push a 1,500-N sofa up a frictionless ramp with a 20° angle. How much force would you have to apply to the sofa? To solve the problem, draw a free body diagram with vectors for **W**, $W_\perp$, $W_{||}$, and the normal force. Show your work below.

| Activity B: | Get the Gizmo ready: | |
|---|---|---|
| **Mechanical advantage and work** | • Select the SIMULATION and CONTROLS tabs.<br>• Click **Reset**. Set the **Angle** to 14°, **Coeff. of friction (μ)** to 0.00, and the **Weight** to 200 N. | *(graph: Applied force, y-axis 200–500)* |
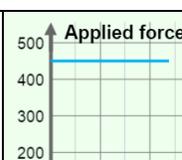
**Question: What determines the helpfulness of an inclined plane?**

1. <u>Observe</u>: Use the Gizmo to find the minimum force needed to push the brick up the ramp.

    A. What is the minimum applied force needed? _____

    B. Select the FREE BODY DIAGRAM tab. What is the magnitude of **W$_{||}$**? _____

    C. How much force would be required to lift the object directly? _____

2. <u>Write a formula</u>: Because less force is needed to lift an object using an inclined plane, it has a **mechanical advantage**. The mechanical advantage is equal to the force needed to lift the object directly divided by the force needed to push the object up the inclined plane.

    Write a formula for mechanical advantage (*MA*) of a ramp in terms of weight (*W*) and *W$_{||}$*:

    *MA =*

3. <u>Calculate</u>: Use your formula to calculate the mechanical advantage of the inclined plane on

    the SIMULATION pane: _____

4. <u>Write a formula</u>: In activity A, you found the formula for *W$_{||}$* in terms of weight (*W*) and the inclined plane's angle (*θ*). Substitute this formula into your mechanical advantage formula and create an equation for mechanical advantage in terms of just the angle (*θ*):

    *MA =*

5. <u>Predict</u>: Based on your formula, how will the mechanical advantage of an inclined plane change if the angle of the plane is increased or decreased? Explain your answer.

    _____

    _____

6. <u>Test</u>: Test your ideas using the Gizmo. What did you find? _____

    _____

**(Activity B continued on next page)**

**Activity B (continued from previous page)**

7. Predict: **Work** is done whenever a force causes an object to move, increasing the object's **mechanical energy**, which is the sum of its potential and kinetic energies. As you have learned, an inclined plane has a mechanical advantage because it makes it easier to lift an object, but do you think an inclined plane helps you to do less work? Explain your answer.

_____

_____

_____

8. Calculate: If the distance moved is in the same direction as the force, the work done by a force equals force × distance. Work can be measured in newton-meters (N·m) or joules (J).

   Select the SIMULATION tab. Click **Reset**. Set the **Angle** to 37°, **Coeff. of friction (μ)** to 0.00, and the **Weight** to 450 N. Move the brick so that its **Height** is exactly 1.00 m.

   A. What force is needed to lift the brick vertically? _____

   B. The top of the ramp is 3.01 m above the ground. How far does the brick need to be

   lifted vertically to reach this height? _____

   C. How much work does a forklift do when lifting the 450-N brick from 1.00 m to 3.01 m

   if it uses the least force necessary for lifting? _____

9. Compare: Use the Gizmo to find the minimum force needed to lift the brick. Set the **Applied force** to this value and click **Play**.

   A. What force is needed to push the brick up the ramp? _____

   B. Select the WORK tab. What is the total distance the brick was pushed to reach the

   top of the ramp? _____

   C. How much work was done on the brick by the car? _____

9. Explain: Why does an inclined plane decrease the amount of force needed to lift the brick but does not decrease the amount of work needing to be done?

_____

_____

_____

| Activity C:<br><br>**Friction and efficiency** | <u>Get the Gizmo ready</u>:<br><br>• Click **Reset**. Set the **Angle** to 37°, **Coeff. of friction (μ)** to 0.25 and the **Weight** to 450 N.<br>• Switch the **External force** to **Off**. |  |
|---|---|---|
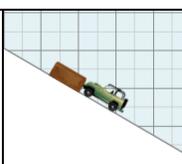
**Introduction:** Wherever two surfaces meet, the force of friction acts to oppose any motion.

**Question: How does friction affect an inclined plane?**

1. <u>Observe</u>: Select the FREE-BODY DIAGRAM tab. The teal blue vector represents friction.

   A. How does the friction vector relate to the direction of movement? _____

   _____

   B. What is the magnitude of the friction force? _____

   C. Divide the magnitude of the friction by the normal force. How does this ratio compare

   to the coefficient of friction? _____

2. <u>Calculate</u>: The **coefficient of friction** ($μ$) is the ratio of the force of friction ($F_f$) to $W_\perp$:

   $$μ = F_f ÷ W_\perp$$

   If $μ = 0.42$ and $W_\perp = 563$ N, what would be the force of friction? _____

3. <u>Explain</u>: Switch the **External force** to **On**. Set the **Applied force** to 400 N and observe the **Friction** vector. Why do you think the vector changed direction?

   A. Why do you think the friction vector changed direction? _____

   _____

   B. What are the magnitudes of $F_f$, $W_{||}$, and the applied force?

   $F_f$: _____        $W_{||}$: _____        Applied force: _____

   C. How is the resultant force calculated, and what is its value? _____

   _____

   D. What happens if the applied force is greater than $W_{||}$ but less than $W_{||} + F_f$? _____

   _____

**(Activity C continued on next page)**

**Activity C (continued from previous page)**

4.  <u>Generalize</u>: When forces work against friction, some energy is wasted as heat. A machine's **efficiency** equals the the mechanical energy transferred to an object divided by the work done by the external force. (Recall that mechanical energy is kinetic energy plus potential energy.)

    *Efficiency = (mechanical energy gained ÷ work done by applied force)*

    A.  What is the efficiency of a frictionless inclined plane? _____

    B.  How could you increase the efficiency of an inclined plane? _____

    _____

5.  <u>Calculate</u>: When a block is pushed up a ramp, it gains both potential energy (*PE*) and kinetic energy (*KE*). Potential energy is equal to weight multiplied by height, while kinetic energy is equal to half of the mass multiplied by the square of the velocity. The unit of energy is the joule (J).

    $$PE = W \cdot h \qquad\qquad KE = \tfrac{1}{2}\, mv^2$$

    Click **Reset**. Check that the **Angle** is 37°, **μ** is 0.25, **Weight** is 450 N, and the **Applied force** is 400 N. On the SIMULATION tab, check that the **Height** of the block is 1.0 m. Click **Play**.

    A.  Multiply the weight of the block by the change in its height to calculate the potential

    energy it gained. (Recall that the block started at a height of 1.0 m.) _____

    B.  Now calculate the kinetic energy gained by the block. (The mass and velocity of the

    block are shown on the SIMULATION pane.) _____

    C.  What is the total mechanical energy gained by the block? _____

    D.  Select the WORK tab. Multiply the applied force by the distance the block moved to

    calculate the work done by the car: _____

    E.  Divide the energy gained by the block by the work done by the car to find the

    efficiency of a 37° ramp with a coefficient of friction of 0.25: _____

6.  <u>Experiment</u>: How do you think the inclined plane's angle and the brick's weight might affect the efficiency of this inclined plane? Experiment with the Gizmo and describe your results.

    _____

    _____

    _____

| | Get the Gizmo ready: | |
|---|---|---|
| **Extension:**<br><br>**The work-energy theorem** | • Click **Reset**. Set the **Angle** to 49°, **Coeff. of friction (μ)** to 0.00, and the **Weight** to 250 N.<br>• Drag the brick so its height is 0.60 m.<br>• Check that the **External force** is **On**. | 3.5  4  4.5 |
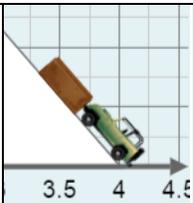
**Introduction:** In previous activities you investigated the work done by the car. In this one you will investigate the work done by the entire system, which includes all forces acting on the brick.

**Question: How does work relate to energy?**

1. <u>Observe</u>: Set the **Applied force** to 289 N by typing this value into the text box to the right of the slider. Press **Play**.

   A. What is the final velocity of the brick? _____

   B. How much kinetic energy did the brick gain? (Recall $KE = \frac{1}{2}mv^2$.) _____

   C. How much potential energy did the brick gain? (Recall $PE = Wh$.) _____


2. <u>Calculate</u>: Select the FREE BODY DIAGRAM tab and the WORK tab.

   A. How far was the brick pushed? _____

   B. How much work did the car do on the brick (round to nearest joule)? _____

   C. What is the net (resultant) force on the brick? _____

   D. Multiply the net force acting on the brick by the distance it traveled to find the total

      work done *by the system* (which includes gravity) on the brick: _____


3. <u>Make two rules</u>: Based on the data in numbers 1 and 2, make two rules: (Fill in the blanks.)

   A. The kinetic energy gained by the brick equals _____

      _____

   B. The potential energy gained by the brick equals _____

      _____


4. <u>Apply</u>: Predict what the final velocity would have been if the car applied 389 N of force.

   _____

**(Extension continued on next page)**

**Extension (continued from previous page)**

5. <u>Test</u>: Check your prediction using the Gizmo. What is the final velocity? _____

6. <u>Conjecture</u>: Do you think either, both, or neither of your rules are true if there is friction?

   _____

   Explain your reasoning: _____

   **_____**

   **_____**

7. <u>Observe</u>: Click **Reset**. Select the FREE BODY DIAGRAM tab and the CONTROLS tab. Set the **Coeff. of friction (μ)** to 0.61. Check that the **Applied force** is still 389 N.

   A.  Why does friction change the **resultant force**? _____

   B.  What is the net force on the brick now? _____

8. <u>Test</u>: Select the CONTROLS tab and click **Play**. Did the two rules you found earlier hold?

   _____

   Show your work in the space below:

9. <u>Conjecture</u>: You should have found that one of the two rules remained true even when there was friction in the system. This rule is known as the **work-energy theorem**: in a mechanical system where various forces act on an object, the change in kinetic energy equals the *net* work done by the system on the object.

   Based on your results, conjecture a rule for potential energy that accounts for friction.

   _____

   _____

10. <u>Apply</u>: A man exerts a force of 450 Newtons on a sled for a distance of 10 meters on a flat

    surface. If the sled started at rest, what is the kinetic energy of the sled now? _____